# Errata to "Unbounded field operators in categorical extensions of conformal nets"

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1. Sec. 2.1. In the proof of Lem. 2.5, the phrase "for any $\eta \in \mathcal{H}_j(\widetilde{J})$ and $\mu \in \mathcal{H}_i(\widetilde{J})$" should be "for any $\eta \in \mathcal{H}_j(J)$ and $\mu \in \mathcal{H}_i(J)$" (i.e., no tilde). Moreover, in the last line of the computation, one should exchange the roles of $\mu$ and $\eta$.

2. Sec. 2.2. In the proof of Thm. 2.10, one should choose $\widetilde{J}$ to be $\widetilde{I}'$ rather than merely clockwise to $\widetilde{I}$. This is because at the end of the proof one chooses $\chi \in \mathcal{H}_0(I')$ and considers $R(\chi, \widetilde{J})$.

3. Sec. 2.4. In the first paragraph of Def. 2.29, one should add the assumption that for each $\mathfrak{a}_1, \ldots, \mathfrak{a}_n \in \mathfrak{H}_i(\widetilde{I})$ and $c_1, \ldots, c_n \in \mathbb{C}$, the operators

$$c_1\mathcal{L}(\mathfrak{a}_1, \widetilde{I}) + \cdots + c_n\mathcal{L}(\mathfrak{a}_n, \widetilde{I}) : \mathcal{H}_k \to \mathcal{H}_i \boxdot \mathcal{H}_k$$
$$c_1\mathcal{R}(\mathfrak{a}_1, \widetilde{I}) + \cdots + c_n\mathcal{R}(\mathfrak{a}_n, \widetilde{I}) : \mathcal{H}_k \to \mathcal{H}_k \boxdot \mathcal{H}_i$$

   are localizable; otherwise Rem. 2.30 may not hold.

   Similarly, in Sec. 2.41 of Sec. 2.7, one show assume that *finite linear combinations of* operators of the form $S\mathcal{L}(\mathfrak{a}_n, \widetilde{I}) \cdots \mathcal{L}(\mathfrak{a}, \widetilde{I})|_{\mathcal{H}_k^\infty}$ are localizable, where $S$ is a homomorphism of $\mathcal{A}$-modules. This is needed in Eq. (2.80) and (2.81).

4. Sec. 2.7. In Def. 2.42, there is a typo: $\widetilde{I}_2$ should be clockwise (rather than anticlockwise) to $\widetilde{I}_1$. Also, the second paragraph can be more precise. The revised version of Def. 2.42 (with only one paragraph) is as follows.

   Let $\mathscr{E}^w_{\text{par}}$ be a categorical partial extension. Given $\mathcal{H}_{i_1}, \ldots, \mathcal{H}_{i_n} \in \mathcal{F}$, $\widetilde{I} \in \widetilde{\mathcal{J}}$, we define $\mathcal{H}_{i_n,\ldots,i_1}(\widetilde{I})$ to be the set of all

$$(\mathfrak{a}_n, \ldots, \mathfrak{a}_1) \in \mathfrak{H}_{i_n}(\widetilde{I}_n) \times \cdots \times \mathfrak{H}_{i_1}(\widetilde{I}_1)$$

   such that $\widetilde{I}_t \subset \widetilde{I}$ and $\mathfrak{a}_t \in \mathfrak{H}_{i_t}(\widetilde{I}_t)$ for each $1 \leqslant t \leqslant n$, and $\widetilde{I}_s$ is clockwise to $\widetilde{I}_t$ whenever $1 \leqslant s < t \leqslant n$.

5. Sec. 3.3. In the paragraph following the proof of Thm. 3.8, when defining $\pi(A)$ using the decomposition $A = UH$, the operator $U$ should be called a partial isometry rather than unitary.

6. Sec. 2.6. In the statement of Thm. 3.37, one should assume that $V, V'$ satisfy condition B, rather than either condition A or condition B. This is because Cor. 3.29 is used at the end of the proof, and it assumes only condition B, not condition A.

7. Sec. 3.7. In the last paragraph of the proof of Thm. 3.39, the formula $\dim_{\mathcal{C}_V}(U) = \dim_{\mathrm{Rep}^0(A)}(U)$ is not correct, and needs to be deleted. In the subsequent sentence, the formula

$$D(\mathcal{C}_V)/D(\mathrm{Rep}^0(A)) = \dim_{\mathrm{Rep}^0(A)}(U)$$

citing [KO02] Thm. 4.5 is also citing incorrectly. The correct formula in that paper is

$$D(\mathcal{C}_V)/D(\mathrm{Rep}^0(A)) = \dim_{\mathcal{C}_V}(U)$$

These two mistakes indeed cancel each other, which again implies $D(\mathcal{C}_V)/D(\mathcal{C}_U) = \dim_{\mathcal{C}_V}(U)$ and hence finishing the proof of Thm. 3.39.

8. Sec. 3.7. In the proof of Lem. 3.47, the last sentence "Thus they also hold for $V_\Lambda \otimes V_L$ and hence for $V_\Lambda$ by theorems 3.38 and 3.39" is a little misleading. The more appropriate one should be "Thus they also hold for $V_\Lambda \otimes V_L$ (by theorems 3.38 and 3.39) and hence for $V_\Lambda$ ".

9. Ch. A. In the beginning paragraph, the statement "$\overline{p_s A^* A} = p_s A^* A p_s$ is bounded and positive" should be "$\overline{q_s A^* A} = q_s A^* A q_s$ is bounded and positive".

10. Ch. A. In the proof of Prop. A.3, the sequence $q_t$ does not converges in SOT to $1$. To fix the issue, one simply replaces $q_t$ by $q_t + E$ where $E$ is the projection onto $\mathrm{Ker} K$.